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PP-wave black holes and the matrix model

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Abstract: We discuss the sizes of a black hole in the M theory pp-wave background, and how the transverse size can be reproduced in the matrix model.

Keywords: Penrose limit and pp-wave background, M(atrix) Theories, Black Holes.

Contents

PP-wave backgrounds, as a limit of AdS and some inner spheres, have proven an interesting place to test ideas of holography and the correspondence between string/M theory in such a background and some supersymmetric gauge theory [1]. In a previous paper [2], we discussed possible correspondence between string states and black hole states in such a background, when the string coupling is fined tuned for a given total oscillation number of the strings. As we shall see, the estimate of the transverse gravitational size of a black hole in that paper is correct, but the longitudinal size given there is incorrect.

In the original paper, the first paper in [1], a matrix model is given for the PP-wave background obtained from the M theory AdS backgrounds. This matrix model was further discussed in [3]. Here we are interested in the question of whether this matrix model can reproduce at least the transverse size of a black hole for a given energy. As we shall argue, indeed it can, provided this model can produce a certain form of interaction between two partons. This form, we will argue, shall come out naturally from a computation in the matrix model, although we will not undertake such a calculation in this paper.

The spirit of the analysis in this paper is that of [4], where Schwarzschild black holes in a flat spacetime is analyzed using matrix theory. The main ingredients in that analysis are to assume that a black hole is a bound state of partons which are completely virialized, and to assume that the minimal uncertainty relation is obeyed by partons.

We will start with an estimation of the transverse size of gravitating point source in the pp-wave background, and also discuss its longitudinal size. Then we turn to the matrix model, give a general analysis of the possible loop corrections to the interaction potential between two partons. We argue that the one loop correction must assume the form

$$V_1 = c\frac{G}{R^3}\frac{v^5}{r^8}\,, \tag{1}$$

where c is a dimensionless numerical coefficient, G is the Newton constant in 11 dimensions, R is the longitudinal radius in the direction x^-, v is the relative velocity between two partons, and r is their transverse separation. Although the power in the velocity dependence sounds a little strange, we will argue that this should be most important term in the one-loop correction, if $\mu r < v$. Happily, as we shall see, this is the right term to reproduce the correct transverse horizon size of a black hole. The more accurate form of (1) appears in (30) and breaks time reversal invariance. This is possible, because the existence of the four form field strength indeed breaks time reversal invariance. We leave the computation of the interaction between two partons to a future work.

1. The gravitational sizes

The metric of the M theory pp-wave background is

$$ds^2 = -4dx^+dx^- - \left[(\frac{\mu^2}{9}\sum_i (x^i)^2 + \frac{\mu^2}{36}\sum_a (x^a)^2\right](dx^+)^2 + \left(dx^A\right)^2 ,$$
$$i = 1,2,3\,, \qquad a = 4\,,\ldots 9\,, \qquad A = i, a\,. \tag{1.1}$$

Since x^+ is taken as our time in the light-cone approach, there is a harmonic potential in each transverse direction, even for a massless particle. This term is the source for a kind of dimensional transmutation in the newtonian potential of a point source.

To see the origin of the dimensional transmutation, let us start with a flat spacetime, where we shall get the standard newtonian potential. To be general, let us work in D dimensional spacetime, the retarded Green's function in the momentum space is simply $1/[(\omega+i\epsilon)^2 - k^2)]$. It proves simplest to work in the light-cone frame to carry out the Green's function in the coordinate space, by first integrating over k_+, and later integrating over the transverse momentum space, the result is

$$G(x) = \frac{1}{4\pi^{D/2}} e^{-\pi Di/4}\theta(x^+)\int_0^\infty dk_- k_-^{\frac{D-4}{2}}\left[e^{-ik_-(x^2-i\epsilon)} - e^{ik_-(x^2+i\epsilon)}\right], \tag{1.2}$$

where $x^+ = x^+x^- - x_\perp^2$, $x_\perp$ is the transverse coordinates. We will drop the subscript $\perp$ in the following expressions.

To estimate the gravitational sizes of a moving particle with longitudinal momentum $p^\pm$, let us use the component of the stress tensor T_{+-} (see [2])

$$T_{+-}(x) = p^-\delta(x^- - ax^+)\delta^{D-2}(x)\,, \tag{1.3}$$

where $a = p^-/p^+$, and we have been loose about the numerical coefficient, since we are interested only in the functional form of the gravitational size. The metric perturbation h_{+-} is then

$$h_{+-} = G\int G(x-y)T_{+-}(y)dy\,, \tag{1.4}$$

again a numerical coefficient is not taken into account. Integration over y^- and y is trivial. Let $\tau = x^+ - y^+$, the metric perturbation is

$$h_{+-} = I_+ - I_-\,, \tag{1.5}$$

with

$$I_+ \sim Gp^-\int_0^\infty dk_- k_-^{\frac{D-4}{2}}\int_0^\infty d\tau \exp\left(-ik_-\left[a\tau^2 + \tau(x^- - ax^+) - x^2 - i\epsilon\right]\right), \tag{1.6}$$

and I_- is given by a similar formula, with $k_-, \epsilon \to -k_-, -\epsilon$. The integration over τ can be carried out approximately, if $k_- a$ is sufficiently large. However, if $x^- - ax^+$ is positive, the arguments in the exponential in (1.6) containing τ are all positive, the integral is damped greatly. This simply implies that the particle at time x^+ has not yet reached the longitudinal position x^-.If $x^- - ax^+ < 0$, the integral over τ can be carried out approximately, we have

$$I_+ \sim Gp^-/\sqrt{a}\int_0^\infty dk_- k_-^{\frac{D-5}{2}} \exp\left(ik_-[x^2 + \frac{(x^- - ax^+)^2}{4a} + i\epsilon]\right). \qquad (1.7)$$

Remember that the integral over τ results in a factor $1/\sqrt{ak_-}$ which is independent of x. Finally, we carry out the integral over k by rotating the contour. Put I_+ and I_- together,

$$h_{+-} \sim G\left(\frac{p^-}{\sqrt{a}}\right)\left[x^2 + \frac{(x^- - ax^+)^2}{4a}\right]^{-\frac{D-3}{2}}. \qquad (1.8)$$

Note that in (1.8), $p^-/\sqrt{a} = \sqrt{p^+p^-} = m$, just the invariant mass of the boosted particle, this must be the case since the strength of the gravitational field must be proportional to the invariant mass in a flat spacetime, due to Lorentz invariance. When we look at near the trajectory of the particle in the longitudinal direction where $x^- - ax^+ = 0$, the gravitational potential assumes the form

$$h_{+-} \sim Gmr^{-(D-3)}, \qquad (1.9)$$

where $r = |x|$. The above is the correct newtonian potential, for the dependence on the transverse direction should be boost invariant. Thus, the gravitational size in the transverse direction is just $r_0^{D-3} \sim Gm$. Set $r = 0$ in (1.8), we obtain the gravitational size in the longitudinal direction $\Delta x^- = \sqrt{a}r_0$. Let $p^\pm = e^{\pm\alpha}m$, α is the boost parameter, we find $\Delta x^- = e^{-\alpha}r_0$, namely, there is the standard Lorentz contraction in the longitudinal direction.

Having discussed the estimate of gravitational sizes of a boosted particle in a flat spacetime, we are ready to discuss these in a pp-wave background. For definitiveness, we work in 11 dimensions. The exact scalar Green's function can be computed in a similar fashion as in [5], we will do it in the appendix. Here for simplicity, we will work with a metric with SO(9) symmetry, namely with a fictitious metric

$$ds^2 = -4dx^+dx^- - \left(\mu^2\sum_a (x^a)^2\right)(dx^+)^2 + (dx^a)^2, \qquad (1.10)$$

where a runs from 1 to 9. To simplify all the formulas in the following, we do the rescaling $x^\mp \to \mu^\pm x^\mp$. We will recover the factor μ in the final result. Although the above metric is quite different from (1.1), the physics captured by it will not be so different. The sizes in two sets of transverse directions in the real space (1.1) may be different, but their dependence on physical parameters such $p^\pm$ and μ will be similar.

The Green's function in the background (1.10) can be computed along the line in [5], and is given by a similar formula as in (1.2). In fact, we simply replace $(x-y)^2$ in that formula by Φ which is

$$\Phi = -4(x^- - y^-)\sin\left(x^+ - y^+\right) - \left[1 - \cos\left(x^+ - y^+\right)\right]\left(x^2 + y^2\right) + (x-y)^2, \qquad (1.11)$$

where x, y are the transverse locations of two spacetime points. Φ maybe interpreted as the invariant spacetime distance between two points in the pp-wave background.

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The component of stress tensor T_{+-} is still given by (1.3), we plug it as well as the Green's function (1.11) into (1.4) and obtain

$$h_{+-} \sim Gp^- \int dk_- k_-^{\frac{D-4}{2}} \int_0^\infty d\tau e^{ik_-(\Phi(\tau)+i\epsilon)} + \cdots , \tag{1.12}$$

where the first term is similar to I_+ in (1.5), and ... denotes the term similar to I_-, $\Phi(\tau)$ is given by

$$\Phi(\tau) = -4\left(x^- - ax^+ + a\tau\right)\sin\tau + x^2\cos\tau \,. \tag{1.13}$$

Thus, the integral over τ in (1.12) is more involved. We will use the stationary method to approximate this integral. The stationary point satisfies

$$\Phi'(\tau) = \left(x^2 + 4a\right)\sin\tau + 4\left(x^- - ax^+ + a\tau\right)\cos\tau = 0\,. \tag{1.14}$$

When $x^- - ax^+ = 0$, on the light-cone trajectory, one stationary point is clear, it is just $\tau = 0$. There is another stationary point, it contributes much less than does the point $\tau = 0$. Near $\tau = 0$, $\Phi(\tau)$ expanded to the second order is

$$\Phi(\tau) = x^2 - \left(4a + \frac{x^2}{2}\right)\tau^2. \tag{1.15}$$

Note that the term proportional to τ^2 is quite different from that in (1.6), now it depends on x^2 as well as on a. When $a \gg x^2$, we go back to (1.6) and obtain a result same as in the flat case. If $x^2 \gg a$, the result is completely different. Integrating out τ as well as k_-, we have

$$h_{+-} \sim Gp^- |x|^{-(D-3)} \left(4a + \frac{x^2}{2}\right)^{-1/2}, \tag{1.16}$$

apparently, if $a \gg x^2$, we obtain the newtonian potential in the flat space. Rescaling back $p^\pm \to \mu^{\pm 1} p^\pm$, the condition is $a \gg \mu^2 x^2$, this is just to say that the harmonic potential in the metric is not important. For $a \ll \mu^2 x^2$, the result is

$$h_{+-} \sim \frac{Gp^-}{\mu |x|^{D-2}}\,, \tag{1.17}$$

this is a result already obtained in [2]. The newtonian potential is modified by a factor $1/(\mu|x|)$. For small x, the result of [2] is incorrect, the reason is that one cannot truncate the Green's function to the first mode in this case as done in that paper.

Thus, the transverse gravitational size is given by

$$r_0^{D-2} = \frac{Gp^-}{\mu}\,. \tag{1.18}$$

For $D = 11$, the case of interest, the exponent is 9 rather than 8. Also, the transverse size no longer depends on the invariant mass, it depends both on the light-cone energy as well as on the parameter μ. We want to note that although the metric (1.1) and that in (1.10) are not Lorentz boost invariant, there is a generalized Lorentz boost invariance

$$\begin{aligned} x^\pm &\to e^{\pm\alpha} x^\pm\,, \qquad p^\pm \to e^{\pm\alpha} p^\pm\,, \\ \mu &\to e^{-\alpha}\mu\,. \end{aligned} \tag{1.19}$$

Formula (1.18) is invariant under this generalized boost.

The other extreme points to look at are when $x^2 = 0$, we want to know the longitudinal size of the gravitating point along the origin in the transverse space. We need to solve the stationary point

$$a \sin\tau + \left(x^- - ax^+ + a\tau\right)\cos\tau = 0\,. \tag{1.20}$$

Let the solution be $\tau_0 > 0$. Near this point

$$\Phi(\tau) = -2a\frac{1+\cos^2\tau_0}{\cos\tau_0}\delta\tau^2 + 4a\frac{\sin^2\tau_0}{\cos\tau_0}\,. \tag{1.21}$$

Using the stationary method, we find

$$h_{+-} \sim Gp^- a^{-\frac{D-2}{2}}|\sin\tau_0|^{-(D-3)}|\cos\tau_0|^{\frac{D-2}{2}}\left(1+\cos^2\tau_0\right)^{-1/2}. \tag{1.22}$$

Apparently, the above result is periodic in $x^- - ax^+$. If $\tau_0(x^- - ax^+)$ is a solution to (1.20), then $\tau_0(x^- - ax^+ - 2\pi na) = \tau_0(x^- - ax^+) + 2\pi n$, and the potential (1.22) is not changed at all. The exact result cannot be exactly periodic in $x^- - ax^+$, since although the function (1.13) is periodic under the double shifts $x^- - ax^+ \to x^- - ax^+ - 2\pi na$, $\tau \to \tau + 2\pi n$, but the range of the integration of τ is also changed. Thus, the gravitational tail in the longitudinal direction should slowly die away.

2. The matrix model analysis

The matrix model proposed in the first reference of [1] is described by the action

$$\begin{aligned}
S &= S_0 + S_m\,,\\
S_0 &= \int dt\,\mathrm{tr}\left(\frac{1}{2R}\left(D_t X^A\right)^2 + \frac{R}{4l_p^6}\left[X^A, X^B\right]^2 + \psi D_t\psi + \frac{iR}{l_p^3}\psi\gamma^A\left[\psi, X^A\right]\right),\\
S_m &= \int dt\,\mathrm{tr}\left(\frac{1}{2R}\left(-\frac{\mu^2}{9}(X^i)^2 - \frac{\mu^2}{36}(X^a)^2\right) - \frac{\mu}{4}\psi\gamma_{123}\psi - \frac{\mu i}{3l_p^3}\epsilon_{ijk}X^iX^jX^k\right).
\end{aligned} \tag{2.1}$$

Again, the matrix model has the global invariance SO(3)$\times$SO(6), not SO(9). For simplicity, in the following dimensional analysis, we assume that the global invariance is SO(9), it is not hard to amend our analysis for the action (2.1), although for our purpose it is not necessary to do this.

To perform a dimensional analysis, let us rescale X and ψ in the following way

$$X \to \frac{l_p^3}{R}X\,, \qquad \psi \to \frac{l_p^3}{R^{3/2}}\psi\,, \tag{2.2}$$

The action after this rescaling takes the form

$$S = \frac{1}{g^2}[S_0 + S_m]\,, \qquad g^2 = \frac{R^3}{l_p^6}\,, \tag{2.3}$$

where the original matrix action S_0 now is independent of l_p and R, and the mass term S_m is also independent of these parameters, and depends only on μ.

It is still possible to classify all the loop corrections into loops weighted by the coupling constant g^2, although the new term S_m does contain an interaction vertex. Consider the 2×2 matrices, describing a system of two partons. We then formally expand the quantum effective action in powers of g^2: $S_{\text{eff}} = \sum_n g^{2(n-1)} S_n$. We are interested in the potential, and the potential has a similar expansion: $V_{\text{eff}} = \sum_n V_n = \sum_n g^{2(n-1)} U_n$. Since the dimension of the coupling constant g^2 is L^{-3}, the dimension of U_n is L^{3n-4}. The rescaled X or r, the separation of the two partons, has a dimension L^{-1}, we can write $U_n = r^{4-3n} f_n$, now f_n is a dimensionless function. If we are interested in the bosonic part only, the dimensionless function f_n is a function of three sets of dimensionless quantities, they are x^i/r, v^i/r^2 and μ/r (now since we are assuming SO(9) symmetry, i runs from 1 to 9). Rescaling back to the original coordinates, we have, in general

$$V_n = R l_p^{3n-6} r^{4-3n} f_n \left(\frac{x^i}{r}, \frac{l_p^3 v^i}{Rr^2}, \frac{l_p^3 \mu}{Rr} \right). \tag{2.4}$$

Of course, the above form is too general to be useful.

To fix the most important term at the one-loop level, without committing concrete calculation starting with action (2.1), we need to make some guess or reasonable physics argument. Here is our reasonable argument. Without the mass term, it is well known that the one-loop bosonic interaction assumes the form $\frac{v^4}{r^7}$, v is the relative velocity between the two partons [6]. There is an additional dimensionful coefficient l_p^9/R^3, of which l_p^9/R^2 can be interpreted as $Gp^+(1)p^+(2)$. the combination $1/(Rr^7)$ can be interpreted as the smearing of the newtonian potential $1/r^8$ over the longitudinal direction (produced by an infinity array of mirror images). Alternatively, this potential can be obtained by considering a D0-brane moving in the Aichelburg-Sexl background. As we already shown, in the pp-wave background, the newtonian potential for large μr is no longer $1/r^8$, but modified to $1/(\mu r^9)$. We expect that the D0-brane interaction should be the smearing of this new potential in the longitudinal direction, thus we should have a factor $1/(R\mu r^8)$ in the potential. It must be also proportional to the Newton constant G and $p^+(1)p^+(2) = 1/R^2$, thus in general we must have

$$V_1 = \frac{G}{R^3 \mu r^8} f\left(v^2, \frac{v^i x^i}{r}\right). \tag{2.5}$$

f is a function of velocity and velocity components. (In the flat background, the loop expansion was analyzed in the second reference of [6].

Is it possible to reproduce the form (2.5) using (2.4) at the one-loop level? A general term of (2.4) for $n = 1$ is

$$R l_p^{-3} r \left[\frac{l_p^3 v^i x^i}{Rr^3} \right]^m \left[\frac{l_p^6 v^2}{R^2 r^4} \right]^n \left[\frac{l_p^3 \mu}{Rr} \right]^p. \tag{2.6}$$

From the dependence of (2.5) on μ, we determine $p = -1$. Now (2.6) becomes

$$\frac{l_p^{3m+6n-6}}{\mu R^{m+2n-2} r^{2m+4n-2}} v^{2n} \left[\frac{v \cdot x}{r} \right]^m. \tag{2.7}$$

If we demand the exponent of l_p is 9, namely $m+2n=5$, we automatically have the exponent of R in the denominator to be 3 and the exponent of r to be 8. Of course, on the dimensional ground, only one of these two exponents is a free parameter, but it is now determined by our general analysis. This analysis does not help us to determine m and n separately, but for our purpose we do not have to know these separately, so we will collectively denote the one-loop potential by

$$V_1 = \frac{Gv^5}{R^3\mu r^8}. \tag{2.8}$$

The fact that the power of v is higher than that in the flat spacetime may have something to do with our previous result on the modified newtonian potential, which is proportional to p^- rather than the invariant mass.

Note that, for small μr and very small v, where r is the characteristic separation between two partons and their distance from the original, the original v^4 interaction is still most important. For this interaction to be smaller than our new interaction (2.8), the condition is $\mu r < v$, namely the relative velocity cannot be too small.

Of course, our general analysis can not replace a direct computation of the one-loop potential in the matrix model, since it is highly nontrivial for possible low order terms to cancel, to yield a result proportional to $1/r^8$.

We now turn to a simple analysis of the ground bound states along the lines of [4]. We assume that a simple black hole is composed of N partons in which each individual parton is essentially a distinguished constituent, and thus satisfies the minimal uncertainty relation

$$\frac{1}{R} v r_0 \sim 1\,, \tag{2.9}$$

or

$$v \sim \frac{R}{r_0}. \tag{2.10}$$

For the matrix model to be effective, v must be small, thus r_0 is much greater than the longitudinal cut-off R.

Next, each parton is subject to interaction with all partons of number N (for large enough N), and the total potential is roughly

$$NV_1 = \frac{NGv^5}{R^3\mu r_0^8}\,. \tag{2.11}$$

It must be the same order of the kinetic energy v^2/R. Using the result (2.10) in $NV_1 \sim v^2/R$, we find

$$r_0^{11} \sim \frac{NGR}{\mu}\,, \tag{2.12}$$

it is certainly invariant under the generalized boost (1.19), or $(\mu, R) \to e^{-\alpha}(\mu, R)$. The above relation can be rewritten as

$$r_0^9 \sim \frac{G}{\mu}\frac{N}{R}\left[\frac{R}{r_0}\right]^2 \sim \frac{Gp^-}{\mu}\,, \tag{2.13}$$

exactly the same formula as (1.18), if we take $D=11$ in that formula. We used I used $P^- \sim Nv^2/R$ and $v \sim R/r_0$ in arriving at (2.13).

We see that indeed the matrix interaction can reproduce the formula for the transverse horizon size of a black hole. In the flat background, N is taken as the entropy of the black hole [4], and there is a general relation $S \sim N \sim r_0 M$, M is the invariant mass of the black hole. In the pp-wave background, there is no boost invariance, so we do not hope in general that the entropy is boost invariant. However, we do have a generalized boost invariance (1.19), and we can define the boost invariant mass by $M = \sqrt{p^+ p^-} \sim \frac{N}{R} v$. Using the uncertainty relation $v \sim R/r_0$, we have

$$N \sim r_0 M \,, \tag{2.14}$$

exactly the same relation as in the flat background. Note that this relation has nothing to do with the detailed formula for r_0, it is a result of the definition of the invariant mass and the uncertainty relation. If N indeed can be regarded as the entropy of the black hole in the pp-wave background, using (2.12) and (2.14), we have

$$r_0^{10} \sim \frac{GMR}{\mu} \,, \qquad S \sim [\frac{GR}{\mu}]^{\frac{1}{10}} M^{\frac{11}{10}} \,, \tag{2.15}$$

in contrast to the relations in the flat background

$$r_0^8 \sim GM \,, \qquad S \sim G^{\frac{1}{8}} M^{\frac{9}{8}} \,. \tag{2.16}$$

A. ??????

We compute the exact Green's function of a massless scalar in the background (1.1). To simplify formulas, assume $\mu = 3$, or alternatively rescale $x^{\pm}$ to absorb μ. The Green's function satisfies

$$\left(-\partial_+ \partial_- - h_{++} \partial_-^2 + \partial^2\right) G(x,y) = \delta^{11}(x-y) \,, \tag{A.1}$$

where h_{++} is the coefficient of $(dx^+)^2$ in (1.1) with $\mu = 3$, ∂^2 is the laplacian in the 9 dimensional transverse space.

Let

$$G\left(x^{\pm} - y^{\pm}, x, y\right) = \int \frac{dk_+ dk_-}{(2\pi)^2} e^{ik_+(x^+ - y^+) + ik_-(x^- - y^-)} G(x,y) \,, \tag{A.2}$$

where x, y are transverse coordinates, then

$$\left(k_+ k_- + h_{++} k_-^2 + \partial^2\right) G(x,y) = \delta^9(x,y) \,. \tag{A.3}$$

Apparently, the Green's function in (A.3) can be expressed as a sum of harmonic eigenstates in the transverse space. For $A = i$, the eigenstates are

$$\phi_{n_i}(x_i) = \left(\frac{\sqrt{|k_-|}}{2^{n_i} n_i! \sqrt{\pi}}\right)^{\frac{1}{2}} H_{n_i}\left(\sqrt{|k_-|} x_i\right) e^{-\frac{1}{2}|k_-| x_i^2} \,. \tag{A.4}$$

For $A = a$, the eigenstates are

$$\phi_{n_a}(x_a) = \left(\frac{\sqrt{|k_-|}}{2^{n_a+1} n_a! \sqrt{\pi}}\right)^{\frac{1}{2}} H_{n_i}\left(\frac{1}{2}\sqrt{|k_-|} x_a\right) e^{-\frac{1}{4}|k_-| x_a^2} \,. \tag{A.5}$$

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The eigenstates of the operator $k_+k_- + h_{++}k_-^2 + \partial^2$ are just products

$$\Phi_\lambda(x) = \prod_i \phi_{n_i}(x_i) \prod_a \phi_{n_a}(x_a)\,. \tag{A.6}$$

Finally, the Green's function of (A.3) is given by

$$G(x,y) = \sum_{\{n_i,n_a\}} \frac{1}{k_+k_- - |k_-| \sum(2n_i+1) + \frac{1}{2}(2n_a+1)} \Phi_\lambda(x)\Phi_\lambda(y)\,. \tag{A.7}$$

Substitute (A.7) into (A.2) and perform the integral over k_+, we obtain a factor $\theta(x_+)$ (for the retarded Green's function) and a product of functions depending on n_i, x_i or n_a, x_a. Following [5], we use the following identity

$$\sum_n \frac{1}{n!} H_n(\sqrt{\omega}x) H_n(\sqrt{\omega}y) (\frac{z}{2})^n = \frac{1}{\sqrt{1-z^2}} \exp(\omega \left(\frac{2xyz - (x^2+y^2)z^2}{1-z^2}\right) \tag{A.8}$$

to obtain a closed form of the integrand in the integral over k_-. In the end, up to a numerical factor, the retarded Green's function can be expressed as

$$G(x,y) = I_+ - I_-\,, \tag{A.9}$$

where

$$\begin{aligned} I_+ &= \cos^3\left(x^+ - y^+\right) \int_0^\infty dk_- k_-^{7/2} e^{ik_-(\Phi+i\epsilon)}\,, \\ \Phi &= 2(x^- - y^-) \sin 2\left(x^+ - y^+\right) + 2xy - \left(x^2+y^2\right)\cos 2\left(x^+ - y^+\right) \\ &\quad + \left(2\tilde{x}\tilde{y} - \left(\tilde{x}^2 + \tilde{y}^2\right)\cos\left(x^+ - y^+\right)\right)\cos\left(x^+ - y^+\right)\,, \end{aligned} \tag{A.10}$$

where $xy = x_ix_i$, $\tilde{x}\tilde{y} = x_ay_a$ and so on. I_- is given by the same formula as (A.10) with the k_- in the exponential replaced by $-k_-$, of course the sign of ϵ must be switched too, to guarantee the convergence of the k_- integral. We can use the Green's function obtained in this appendix to repeat the analysis in the main text.

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